Cambridge International A Level

**Cambridge International Examinations**
Cambridge International Advanced Level

**FURTHER MATHEMATICS** **9231/23**

Paper 2 **May/June 2017**

MARK SCHEME

Maximum Mark: 100

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

**Mark Scheme Notes**

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

## Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | *EITHER:*<br>$T \times a = (\frac{1}{2} ma^2)\, \mathrm{d}^2\theta / \mathrm{d}t^2$ *or* $(\frac{1}{2} ma)\, \mathrm{d}^2x / \mathrm{d}t^2$ | **(B1** | Find eqn of motion for disc |
| | $3mg - 0{\cdot}9mg - T = 3m \times a\, \mathrm{d}^2\theta / \mathrm{d}t^2$ *or* $3m\, \mathrm{d}^2x / \mathrm{d}t^2$ | **M1 A1)** | Find eqn of motion for block |
| | *OR:*<br>$Ta\theta = \frac{1}{2} (\frac{1}{2} ma^2) (\mathrm{d}\theta / \mathrm{d}t)^2$ | **(B1** | Find eqn. of energy for disc (in terms of $\theta$ or $x$) |
| | $(3mg - 0{\cdot}9mg - T)\, a\theta = \frac{1}{2}\, 3m\, (a\, \mathrm{d}\theta / \mathrm{d}t)^2$ | **M1 A1)** | Find eqn of energy for block (in terms of $\theta$ or $x$) |
| | $6T = 2{\cdot}1mg - T$, $T = 0{\cdot}3\, mg$ | **M1 A1** | Combine two eqns to find $T$ |
| | **Total:** | **5** | |
| 2(i) | $OL/OM = \frac{3}{4}$, $OM = 2$ [m] | **M1 A1** | Find $OM$ by equating ratios of distances and acceln. |
| | **Total:** | **2** | |
| 2(ii) | *EITHER:*<br>$\omega\, t_{LM} = \sin^{-1} (OL/2{\cdot}5) + \sin^{-1}(OM/2{\cdot}5)$<br>*or* $\omega\, t_{OL} = \sin^{-1} (OL/2{\cdot}5)$, $\omega\, (2 - t_{OL}) = \sin^{-1} OM/2{\cdot}5)$<br>so $2\omega = \sin^{-1} 0{\cdot}6 + \sin^{-1} 0{\cdot}8$ [= 0·6435 + 0·9273] | **(*M1 A1)** | Find eqn. (AEF) for $\omega$, for example using $x = a \sin \omega t$ |
| | *OR:*<br>$\omega\, t_{LM} = \cos^{-1} (-OM/2{\cdot}5) - \cos^{-1}(OL/2{\cdot}5)$<br>*or* $\omega\, t_{AL} = \cos^{-1} (OL/2{\cdot}5)$, $\omega\, (2 + t_{AL}) = \cos^{-1} (-OM/2{\cdot}5)$<br>so $2\omega = \cos^{-1} (-0{\cdot}8) - \cos^{-1} 0{\cdot}6$ [= 2·498 – 0·927]<br>*or* $\pi - \cos^{-1} 0{\cdot}8 - \cos^{-1} 0{\cdot}6$ | **(*M1 A1)** | *or* $x = a \cos \omega t$<br>($A$ is at 2·5 from $O$, near $L$) |
| | $\omega = \pi / 4$ *or* 0·785 (M1 dep *M1) | **DM1 A1** | Simplify to find $\omega$ (may be implied by $T$) |
| | $T = 2\pi / \omega = 8$ [s] | **B1 FT** | Find period $T$ (FT on $\omega$) |
| | **Total:** | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(iii) | $v_L = \omega\sqrt{(2{\cdot}5^2 - 1{\cdot}5^2)} = 2\omega = \pi / 2$ *or* $1{\cdot}57$ [m s$^{-1}$] | **M1 A1** | Find speed $v_L$ at $L$ |
| | **Total:** | **2** | |
| 3(i) | $mv_A + mv_B = mu$ (AEF) | ***M1** | Use conservation of momentum (allow $v_A + v_B = u$) |
| | $v_B - v_A = \frac{2}{3}\,u$ | ***M1** | Use Newton's restitution law (consistent LHS signs) |
| | $v_B = 5u/6$ | **A1** | Combine to find $v_B$ |
| | $w_B = \frac{1}{3}\,v_B = 5u/18$ AG | **B1** | Verify speed $w_B$ of $B$ after collision with wall (ignore sign) |
| | **Total:** | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(ii) | $v_A = u / 6$ | **DA1** | Find $v_A$ (dependent on above ***M1 *M1**) |
| | *EITHER:*<br>$(d – x) / v_A = d / v_B + x / w_B$ (AEF) | **(M1 A1** | *EITHER:* Equate times in terms of reqd. distance $x$ |
| | $6(d – x) = 1{\cdot}2\ d + 3{\cdot}6\ x$ | **M1 A1)** | Substitute for speeds to formulate an eqn. in $x$ |
| | *OR:*<br>$x_A = (d/v_B)\ v_A = (6d/5u)\ u/6 = 0{\cdot}2\ d$ | **(M1** | *OR:* Find dist. $x_A$ moved by $A$ when $B$ reaches wall |
| | $t_2 = (0{\cdot}8\ d) / (v_A + w_B) = 9d/5u$ | **M1 A1** | Find remaining time $t_2$ |
| | $y_A = v_A\ t_2 = 0{\cdot}3\ d$ *or* $y_B = w_B\ t_2 = 0{\cdot}5\ d$ | **A1)** | Find remaining distance moved by $A$ or $B$ |
| | *OR2:*<br>$x_A = (d/v_B)\ v_A = (6d/5u)\ u/6 = 0{\cdot}2\ d$ | **(M1** | *OR2:* Find dist. $x_A$ moved by $A$ when $B$ reaches wall |
| | $(0{\cdot}8\ d – x) / v_A = x / w_B$ *or* $0{\cdot}8\ d/(v_A + w_B) = x/w_B$ | **M1 A1** | Equate remaining times to formulate an eqn. in $x$ |
| | $4{\cdot}8\ d – 6\ x = 3{\cdot}6\ x$ *or* $1{\cdot}8\ d = 3{\cdot}6\ x$ | **A1)** | |
| | $x = ½\ d$ | **A1** | Find $x$ |
| | **Total:** | **6** | |
| 4(i) | $T \times 3a \sin\theta = W \times 1{\cdot}5a \cos 2\theta$ | **M1 A1** | Take moments for rod about $A$ |
| | $T = 7W/30$ *or* $0{\cdot}233W$ | **A1** | Find tension $T$ |
| | **Total:** | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(ii) | $OB = (5a - 3a\cos 2\theta) / \cos\theta = 26a/5$ *or* $5{\cdot}2a$ | **M1 A1** | Find length $OB$ of string |
| | $T = \lambda(OB - 4a)/4a$ | **M1** | Find modulus $\lambda$ using Hooke's Law |
| | $= \lambda(6a/5)/4a = 3\lambda/10$, $\lambda = 7W/9$ *or* $0{\cdot}778W$ | **A1** | |
| | **Total:** | **4** | |
| 4(iii) | $X = T\cos\theta$ [$= 14W/75$ *or* $0{\cdot}187W$] | **M1** | Find horizontal component $X$ of force at $A$ |
| | $Y = T\sin\theta + W$ [$= 57W/50$ *or* $1{\cdot}14W$]<br>$\phi = \tan^{-1}(Y/X) = \tan^{-1}(171/28)$ | **M1** | Find vertical component $Y$ of force at $A$<br>Find angle $\phi$ which force at $A$ makes with horizontal |
| | $= 80{\cdot}7°$ *or* $1{\cdot}41$ radians | **B1** | |
| | **Total:** | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(i) | $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga(\cos\alpha + \sin\alpha)$<br>$v^2 = u^2 + 2ag(4/5 + 3/5)$<br>$= u^2 + 14ag/5$ AG | **M1 A1** | Verify $v$ by conservation of energy (**A0** if no $m$) |
| | **Total:** | **2** | |
| 5(ii) | $T_P = mu^2/a - mg\cos\alpha$ | **B1** | Find tension $T_P$ at $P$ by using $F = ma$ |
| | $T_Q = mv^2/a + mg\sin\alpha$ | **B1** | Find tension $T_Q$ at $Q$ by using $F = ma$ |
| | $v^2 = 2u^2 - ag(2\cos\alpha + \sin\alpha) = 2u^2 - 11ag/5$ (AEF) | **M1 A1** | Relate $u^2$, $v^2$ using $T_Q = 2T_P$ (**A0** if reqd. eqn omitted) |
| | $u = \sqrt{(5ag)}$ *or* $2{\cdot}24\sqrt{(ag)}$ [$v^2 = 39ag/5$] | **A1** | Eliminate $v^2$ to find $u$ |
| | **Total:** | **5** | |
| 5(iii) | $T_{min} + mg = mV^2/a$ | **B1** | Find tension $T_{min}$ at top from $F = ma$ radially |
| | $\frac{1}{2}mV^2 = \frac{1}{2}mu^2 - mga(1 - \cos\alpha)$<br>*or* $\frac{1}{2}mv^2 - mga(1 + \sin\alpha)$ | **M1** | Find $V^2$ at top by conservation of energy (**A0** if no $m$) |
| | [$V^2 = (5 - 2/5)ag = (39/5 - 16/5)ag = 23ag/5$]<br>$T_{min} = 23mg/5 - mg = 18mg/5$ *or* $3{\cdot}6mg$ | **A1** | Combine to find $T_{min}$ |
| | **Total:** | **3** | |
| 6 | $(10 - 4^2/N + 102 - 8^2/2N) / (N + 2N - 2)$ (AEF) | **M1 A1** | State or find expression for pooled estimate of $\sigma^2$ (confusing biased/unbiased estimates may still earn **M1**) |
| | $112 - 48/N = 10(3N - 2)$, $15N^2 - 66N + 24 = 0$ | **M1 A1** | Equate to 10 and rearrange as quadratic |
| | $N = (66 \pm 54) / 30 = 4$ | **A1** | Solve quadratic for $N$, rejecting root $0{\cdot}4$ |
| | **Total:** | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(i) | $r = \sqrt{(0{\cdot}46 \times 0{\cdot}93)} = 0{\cdot}654$ | **M1 A1** | Find correlation coefficient $r$ |
| | **Total:** | **2** | |
| 7(ii) | $H_0$: $\rho = 0$, $H_1$: $\rho \neq 0$ | **B1** | State both hypotheses (**B0** for $r$ …) |
| | $r_{12,\ 5\%} = 0{\cdot}576$ | **B1** | State or use correct tabular two-tail $r$-value |
| | Accept $H_1$ if $\|r\|$ > tab. value (AEF) | **M1** | State or imply valid method for conclusion (**M0** if $r$ or tab. value has magnitude > 1) |
| | 0·654 [*or* 0·65] > 0·576 so there is non-zero correlation | **A1** | Correct conclusion (AEF) |
| | **Total:** | **4** | |
| 8 | $\bar{x} = 28{\cdot}32$ *and* $\bar{y} = 22{\cdot}2$ | **B1** | Find both sample means |
| | $s_A^2 = (41\ 100 - 1416^2/50) / 49$ *and* $s_B^2 = (20\ 140 - 888^2/40) / 39$ | **M1** | Estimate both population variances |
| | $s_A^2 = 20{\cdot}39$ *and* $s_B^2 = 164/15$ *or* 10·93 (to 3 s.f.) | **A1** | (allow biased here: 19·98 *and* 10·66) |
| | *EITHER:* $s^2 = s_A^2/50 + s_B^2/40 = 0{\cdot}681$ *or* $0{\cdot}825^2$ (19·98/50 + 10·66/40 = 0·666 is M1 A0, max 8/9) | **(M1 A1** | *EITHER:* Estimate combined variance (if biased values used wrongly here, giving $zs$ = 1·60, only this A1 is lost) |
| | $\bar{x} - \bar{y} \pm z\,s$ | **M1** | Find confidence interval for difference |
| | $z_{0.975} = 1{\cdot}96$ | **A1** | Use appropriate tabular value |
| | $z\,s = 1{\cdot}62$ | **A1** | Evaluate semi-interval length |
| | 6·12 ± 1·62 *or* [4·50, 7·74] | **A1)** | State confidence interval (in either form) |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| | *OR:*<br>Assume equal [population] variances<br>$s^2 = (49\ s_A^2 + 39\ s_B^2) / 88$<br>*or* $(41\ 100 – 1416^2/50 + 20\ 140 – 888^2/40) / 88$ | **(B1** | *OR:*<br>State assumption<br>Find pooled estimate of common variance<br>(**M1 A1** for $s_A^2$ and $s_B^2$ may be implied here) |
| | $= 16{\cdot}2$ *or* $4{\cdot}02^2$ | **B1** | |
| | $\bar{x} - \bar{y} \pm z\ s\ \sqrt{(1/50 + 1/40)}$ | **M1** | Find confidence interval for difference |
| | $z_{0.975} = 1{\cdot}96$ | **A1** | Use appropriate tabular *z*-value<br>(or appropriate *t*-value from calculator or interpolation) |
| | ($t_{88,\ 0.975} = 1{\cdot}9873$ *or* $1{\cdot}99$)<br>$1{\cdot}67$ ($1{\cdot}70$) | **A1** | Evaluate semi-interval length |
| | $6{\cdot}12 \pm 1{\cdot}67$ *or* [4·45, 7·79]<br>($6{\cdot}12 \pm 1{\cdot}70$ *or* [4·42, 7·82]) | **A1)** | Evaluate confidence interval (in either form) |
| | **Total:** | **9** | |
| 9(i) | $(a / \ln 2)\ [-e^{-x \ln 2}]_0^\infty = a / \ln 2$ so $a = \ln 2$ *or* $0{\cdot}693$ | **M1 A1** | State *a* or find *a* by equating $\int_0^\infty f(x)\ dx$ to 1 |
| | **Total:** | **2** | |
| 9(ii) | $E(X) = 1 / \ln 2$ *or* $1{\cdot}44$ | **B1** | State or find $E(X)$ |
| | **Total:** | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(iii) | $F(Q) = 1 - e^{-Q \ln 2} = ¼$ *or* ¾ | **M1** | Formulate equation for either quartile value $Q$ |
| | $Q_1 = (\ln 4/3) / (\ln 2)$ [= 0·415 ] (AEF) | **A1** | Find one [lower] quartile $Q_1$ |
| | $Q_3 = (\ln 4) / (\ln 2)$ [= 2] (AEF) | **A1** | Find other [upper] quartile $Q_3$ |
| | $Q_3 - Q_1$ [= (ln 3) / (ln 2)] = 1·58 [*or* 1·59] | **A1** | Find interquartile range (allow $Q_1 - Q_3$) |
| | **Total:** | **4** | |
| 9(iv) | *EITHER:*<br>$G(y) = P(Y < y) = P(2^X < y)$<br>$= P(X < (\ln y) / (\ln 2))$<br>$= F((\ln y) / (\ln 2))$ *or* $F(\log_2 y)$ (AEF) | **(M1 A1** | Find or state $G(y)$ for $x \geqslant 0$ from $Y = 2^X$<br>(allow < or ≤ throughout) |
| | $= 1 - e^{-\ln y}$ *or* $1 - 1/y$ | **A1)** | |
| | *OR:*<br>Use $x = (\ln y) / (\ln 2)$ to find *both* | **(M1** | Find $f(x)$ *and* $dx/dy$ for use in $g(y) = f(x) \times \lvert dx/dy \rvert$ |
| | $f(x) = (\ln 2)\, e^{-x \ln 2} = (\ln 2)\, e^{-\ln y} = (1/y) \ln 2$ | **A1** | |
| | *and* $dx/dy = 1 / (y \ln 2)$ | **A1)** | |
| | $g(y)$ [= $G'(y)$] = $1/y^2$ | **A1** | Find $g(y)$ in simplest form |
| | for $y \geqslant 1$ [$g(y) = 0$ otherwise] | **A1** | State corresponding range of $y$ for $G(y)$ or $g(y)$ |
| | **Total:** | **5** | |
| 10(i) | $\bar{x} = (1/100)\, \Sigma\, x\, f(x) = 325/100 = 3·25$ AG | **B1** | Verify given mean (**B0** for $\bar{x} = 325/100 = 3·25$) |
| | **Total:** | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(ii) | $E_3 = 100\ \lambda^3\ e^{-\lambda}/3!$ with $\lambda = 3{\cdot}25$ | **M1 A1** | State expression for reqd. expected value $E_3$ (**M1** for $E_3$ *or* $E_6$) |
| | $E_6 = 100\ \lambda^6\ e^{-\lambda}/6! = 6{\cdot}35$ | **A1** | Find exp. value $E_6$ |
| | $E_{\geqslant 7} = 100 - \Sigma_0^6\ E_i = 4{\cdot}77$ | **A1** | Find exp. value $E_{\geqslant 7}$ |
| | **Total:** | **4** | |
| 10(iii) | $H_0$: Distribution fits data (AEF) | **B1** | State (at least) null hypothesis in full |
| | $O_i$: 13 18 26 20 16 7<br>$E_i$: 16·48 20·48 22·18 18·02 11·72 11·12 | **M1FT** | Combine values consistent with all exp. values $\geqslant 5$ (FT on $E_6$ and $E_{\geqslant 7}$) |
| | $\chi^2 = 0{\cdot}735 + 0{\cdot}300 + 0{\cdot}658 + 0{\cdot}218 + 1{\cdot}563 + 1{\cdot}527$ | **M1** | Find $\chi^2$ |
| | $= 5{\cdot}00$ | **A1** | |
| | No. $n$ of cells: 8 7 6 5 4<br>$\chi^2_{n-2,\,0.95}$: 12·59 11·07 9·488 7·815 5·991 | **B1FT** | State or use consistent tabular value $\chi^2_{n-2,\,0.95}$ (to 3 s.f.) [FT on number, $n$, of cells used to find $\chi^2$] |
| | Accept $H_0$ if $\chi^2$ < tabular value (AEF)<br>5·00 [± 0·1] < 9·49 so distn. fits [data] | **M1** | State or imply valid method for conclusion<br>Conclusion (requires both values correct) |
| | *or* distn. is a suitable model (AEF) | **A1** | |
| | **Total:** | **7** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 11(a)(i) | $I_{AB} = \frac{1}{3}\ 8m\ (3a/2)^2 + 8m\ (a/2)^2$<br>*or* $8m\ (3a)^2/12 + 8m\ (a/2)^2$ [$= 8\ ma^2$] | **M1 A1** | Find or state MI of rod $AB$ about axis $l$ |
| | $I_{shell} = \frac{2}{3}\ ma^2 + m\ (2a)^2$ [$= (14/3)\ ma^2$] | **M1** | Find MI of shell about axis $l$ |
| | $I_{sphere} = (2/5)\ (3m/2)a^2 + (3m/2)\ (2a)^2$ [$= (33/5)\ ma^2$] | **M1 A1** | Find MI of sphere about axis $l$ |
| | $I = (8 + 14/3 + 33/5)\ ma^2 = (289/15)\ ma^2$ AG | **A1** | Verify MI of object about axis $l$ |
| | **Total:** | **6** | |
| 11(a)(ii) | $\frac{1}{2}\ I\ \omega^2 = (5mg/2)\ 2a\ (1 - \cos\alpha) - 8mg\ (a/2)\ (1 - \cos\alpha)$<br>*or* $((21/2)mg \times 2a/21)\ (1 - \cos\alpha)$ | **M1 A1** | Find $\omega^2$ or angular speed $\omega$ when $CA$ vertical by energy |
| | $= mga\ (1 - \cos\alpha) = 5mga/6$ | **A1** | |
| | $\omega^2 = 25g/289a$ *or* $0{\cdot}0865\ g/a$<br>*or* $\omega = (5/17)\ \sqrt{(g/a)}$ *or* $0{\cdot}294\ \sqrt{(g/a)}$ | **A1** | |
| | $v_{max} = 2a\omega = 2a\ \sqrt{(25g/289a)}$<br>$= \sqrt{(100ag/289)}$ *or* $(10/17)\sqrt{(ag)}$ | **M1** | Find maximum speed $v_{max}$ of $O$ from $r\omega$ |
| | *or* $0{\cdot}588\sqrt{(ag)}$ *or* $1{\cdot}86\sqrt{a}$ (AEF) | **A1** | (**A1** requires some simplification) |
| | **Total:** | **6** | |
| 11(b)(i) | e.g.<br>$S_{xy} = 4510{\cdot}99 - 191 \times 188{\cdot}8/8 = 3{\cdot}39$ *or* $0{\cdot}424$<br>$S_{xx} = 4564{\cdot}46 - 191^2/8 = 4{\cdot}335$ *or* $0{\cdot}542$<br>$b = S_{xy} / S_{xx} = 3{\cdot}39/4{\cdot}335 = 0{\cdot}782$ | **M1 A1** | Find reqd. values |
| | $(y - 188{\cdot}8/8) = b\ (x - 191/8)$<br>$(y - 23{\cdot}6) = 0{\cdot}782\ (x - 23{\cdot}875)$, $y = 0{\cdot}782x + 4{\cdot}93$ | **M1 A1** | Find gradient $b$ in $y - \bar{y} = b\ (x - \bar{x})$<br>and hence eqn. of regression line (may be implied by writing $y = a + bx$ and finding $a$, $b$) |
| | **Total:** | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 11(b)(ii) | $H_0$: $\mu_x - \mu_y = 0{\cdot}2$, $H_1$: $\mu_x - \mu_y > 0{\cdot}2$ (AEF) | **B1** | State both hypotheses (**B0** for $\bar{x}$ … ) |
| | $d_i$: 0·3 0·2 0 0·6 0·3 0·5 0·2 0·1 | **M1** | Consider differences $d_i$, e.g. $x_i - y_i$ |
| | $\bar{d} = 2{\cdot}2 / 8 = 0{\cdot}275$ | **B1** | Find sample mean |
| | $s^2 = (0{\cdot}88 - 2{\cdot}2^2/8) / 7$ | **M1** | Estimate population variance<br>(allow biased here: [11/320 *or* 0·0344 *or* $0{\cdot}185^2$ ]) |
| | [ = 11/280 *or* 0·0393 *or* $0{\cdot}198^2$]<br>$t_{7,\,0.9} = 1{\cdot}41[5]$ | **B1** | State or use correct tabular *t*-value |
| | $t = (\bar{d} - 0{\cdot}2) / (s/\sqrt{8}) = 1{\cdot}07$ | **M1 A1** | Find value of *t*<br>(or compare $\bar{d} - 0{\cdot}2 = 0{\cdot}075$ with $t_{7,\,0.9}\, s/\sqrt{8} = 0{\cdot}099$) |
| | [Accept $H_0$:] No evidence for coach's belief (AEF) | **B1 FT** | Consistent conclusion (FT on both *t*-values)<br>SR Wrong (hypothesis) test can earn only<br>**B1** for hypotheses<br>**B1**FT for conclusion (max 2/8) |
| | **Total:** | **8** | |